How to design and tolerance with GRADIUM® glass

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ABSTRACT

Designing with axial-gradient materials can be a complicated task. The difficulties range from the speed of ray-tracing codes and the mechanics of specifying the material and appropriate variables to selecting the best gradient and orientation from a set of fixed profiles. We propose a simple methodology for designing with axial-gradient glasses in modern ray-trace codes. The first step is to determine locations where the gradient can be useful. This decision may be made by probing a design with aspheres or by analysis of the design to decide what needs to be corrected. The second step is to modify the design for appropriate base materials. GRADIUM® lenses act as correctors in the optical system and the first-order optical properties still must be controlled in the normal manner. The third step is to design the optimal gradient for the application. While the designer will only have the option of designing the gradient for actual use in a very limited set of cases, understanding the shape of the ideal gradient will allow the designer to select the profile and orientation that most closely matches the ideal. Then the designer can work on best implementing the design and fine-tuning the design.

Tolerancing and preparation of the GRADIUM lens print require only a few additional steps and understanding of how the material is fabricated. For example, the maximum profile thickness is nominal and may not correspond to the physical dimensions of a blank, such as when a blank is pre-thinned.

Keywords: Gradient-index, optical design, tolerancing, lens manufacture

1. INTRODUCTION

Axial-gradient technology has become an appropriate choice for mainstream optical design because of the success of LigthPath's efforts to improve the process to the point that repeatability and quality are excellent. However, despite the advantages to the optical design, there are many impediments to the designer. In particular, designing with inhomogeneous materials requires more significant computational resources. In addition, because relatively few designers have experience with these materials, the empirical experience base ("bag of tricks") is smaller. Our goal in this paper is to provide examples of techniques and design principles that have been useful to experienced designers as well as to provide information that will be helpful when ordering lenses or material from LightPath.

2. DESIGNING GRADIUM INTO A SYSTEM

2.1. How GRADIUM affects an optical system

The single most important realization—we shall be pedantic about it—is that *the axial gradient in a GRADIUM lens represents correction degrees of freedom.* An optical system must still have the first order properties satisfied. These are the same first-order properties that would be calculated if the system were designed with all homogeneous materials. The GRADIUM elements correct aberrations and allow the design to perform better, do something that was not possible (because of, for example, size constraints) or to be realized with fewer elements. The particular results depend very much on the goals of the project.

A classic application of GRADIUM lenses is for the correction of spherical aberration. We see this property used in singlets and doublets designed for the LightPath catalog. In many applications—zoom lenses, riflescopes, and others—we have seen that using a GRADIUM lens near the stop is very effective. In some cases, it provides all the correction that is needed in the system to meet the new specifications. This is in part due to the fact that certain other issues, such as color correction or element manufacturability, may result in many lenses (and therefore many degrees of freedom) in the optical system, reducing the correction that must be done by the GRADIUM element(s).

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An axial gradient is not precisely equivalent to an asphere. If we consider Snell's law and a monochromatic application, then it is clear that, in order to get the desired aiming of rays passing through a surface, both n and θ are equally effective variables; a gradient and an arbitrary asphere are equivalent since modifications of the index profile or the surface shape (and hence surface normal angle, θ) yield the same effect. There is one important restriction, however. Because of practical limitations on refractive indices of optical materials (which we must somehow be joined to form a gradient), if the element is large enough, there are simple aspheres (such as strong conics) that cannot be duplicated with an axial gradient. (There are other gradient/surface forms that may, however, be useful.) This is because the Δn required by such designs can exceed the total index range of known materials.

When the system becomes polychromatic, the strict equivalency between aspheres and axial-gradients is broken because the refractive index gradient varies in a manner that is more complicated than the simple single index of refraction changes seen in the asphere. The asphere always has the same surface form and a single wavelength-dependent variable. The changes in the gradient are much like allowing the shape of the asphere to vary as a function of wavelength also. As will be noted later, this difference is an advantage of gradients. The precise properties depend upon the orientation of the glass line.

There may be times when a GRADIUM element does not improve the design. There are times when an asphere does not improve a design. If the GRADIUM element is used in lieu of an element that contributes little to the monochromatic aberrations, it will do little. If the GRADIUM element is used in such a way as to expose a relatively small index variation, its performance will not be significantly different than a homogeneous element. If a design is severely stressed and the GRADIUM element is not being used to correct the dominant aberration, the improvement will be small. The designer's skill in finding the right use for the gradient is crucial to successful GRADIUM designs.

2.2. Design Methodology

There are several approaches to effecting a design. In the best of all worlds, the designer would have enough experience to move quickly to a solution. However, this is too often not the case. Thus we present a methodology that can be used to work to a design solution. White papers from the respective software vendors will lead the user through the mechanics of implementation.

The first problem is to identify appropriate locations for GRADIUM elements. Seidel coefficients for each surface can be a useful starting point to see where correction is being effected. Unfortunately, CODE V is the only major commercial design code capable "out of the box" (at the time of this writing) of calculating the third-order Seidel coefficients for inhomogeneous materials (although it does not calculate chromatic aberration coefficients). Richard Pfisterer has written an OSLO macro capable of doing the calculations for an axial gradient (the transfer contributions are calculated assuming a linear gradient; they are usually negligible unless the marginal ray angles are very large). This macro is given in the appendix (§ 7.1).

Because of the limited ability to use Seidel coefficients, the first recommended step is to examine the ray fans to identify the problems and probe the design with simple aspheres of the form $a\rho^4$ to determine likely locations to affect the monochromatic aberrations. Such aspheres are recommended because the ray tracing is more computationally efficient and the *a* coefficient is proportional to the (linear) gradient slope.

Once candidate locations have been identified, we suggest determining the GRADIUM glass family that will be used in each location. There are still only a limited number (as of this writing, two) of GRADIUM glass families available, as shown in Figure 1. Both are high index glass families and differ in number of profiles, index variation, and dispersion. The GSF family is flint with properties similar to standard SF glasses. The GLAK family is a crown with LaK properties. Then the second step is to modify the base design to accommodate the new materials, particularly since color and other first-order properties (such as field curvature) will be affected by the change. During this step, sometimes it is useful to use an asphere made from the appropriate base material. This may allow the design to correct for aberrations that are expected to be present in the final form. Particularly if color correction is an issue, it is important to realize that the design merit function is

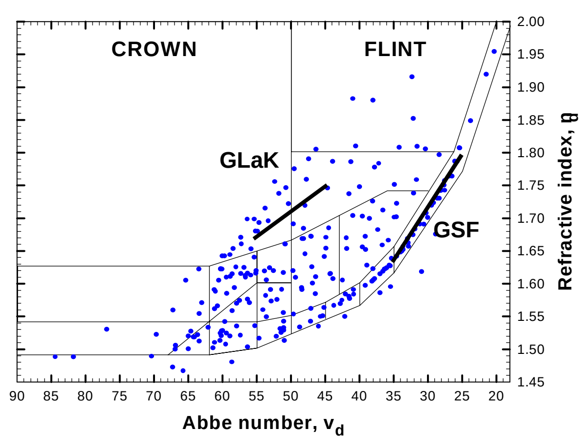

Figure 1. Glass map showing GSF and GLAK lines.

probably going to increase as the design is moved from one minimum to another.

The third step is to replace the asphere with the best gradient equivalent. Earlier the decision should have been made which glass family was to be used, so the asphere will be replaced by an arbitrary gradient (I would only use linear—sometimes quadratic also—terms to model the gradient). The software should call the appropriate dispersion model (GSF or GLAK). An appropriate base index should be used. Since the current materials are used, an n_0=1.7 default is reasonable. When the ideal gradients are plotted, the designer will know whether to use a positive or a negative gradient slope. The sign of the Seidel gradient coefficients can also help the designer determine of the gradient is oriented correctly. By comparing the designed gradient with available materials, the designer will be able to select the closest real material. Also, if the ideal gradient is dramatically different from anything available, the designer will be alerted to the fact that it will be very difficult to apply the gradient in this situation. This step is analogous to designing with model glasses.

The fourth and final step is to substitute in the actual gradient and finish fine-tuning the design. One word of caution here—not all software packages effectively bound the gradient. LightPath's GRADIUM materials are available only in specific thicknesses and the final lens *must* come completely from within the defined material thickness. Some of the codes do not automatically ensure that only valid Δz, thickness, and radius combinations are considered. Besides the material not being available, the normalized polynomial that describes GRADIUM materials may return nonsensical values if used out of the defined range. ZEMAX, for example, does a good job of keeping the lens within the GRADIUM blank. However, because of a feature used for what-if analyses, it can consider positions outside of the blank. In this case the derivatives change abruptly and can stagnate the optimizer and produce erroneous results. A quick look at the profile plot will show the problem (that the optimizer wants to be at the top or the bottom of a profile) and it can be fixed. There are other techniques which help ZEMAX track its position. Similar tips for ZEMAX and the other codes (CODE V, OSLO and SIGMA) are found in materials from the vendors.

2.3. Example: Double Gauss lens

2.3.1. Mandler reference design

The design data are taken directly from the literature[1] with the only exception that the speed has been reduced to f/2.6. This design has been used as a Double Gauss reference for a radial grin objective design.[2] The Double Gauss presented in this section is a direct comparison to the Pfisterer GRADIUM derivative (§ 2.3.3) with identical data:

EFL	=	50mm
F-Number	=	2.6
FOV	=	+/- 20 deg

Figures 2-4 show the layout, ray fan and MTF plots.

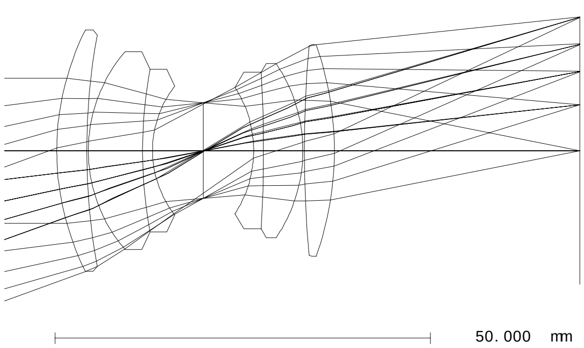

Figure 2. Mandler layout.

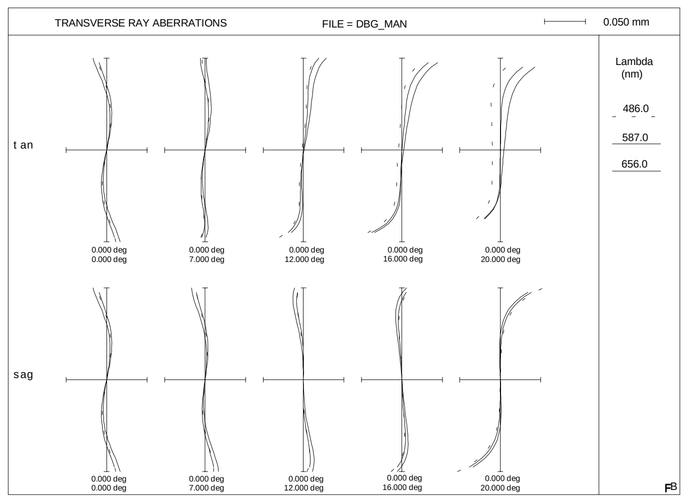

Figure 3. Mandler ray fans.

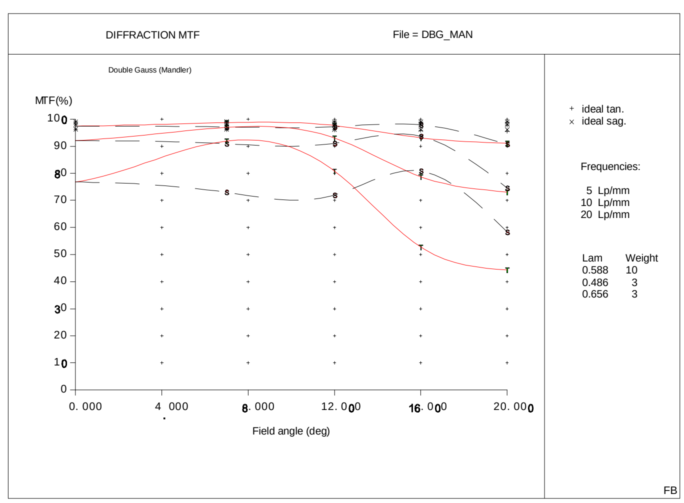

Figure 4. Mandler diffraction MTF versus field.

2.3.2. Pfisterer design

Richard Pfisterer took the Mandler design and reduced his double Gauss form to its essence: a pair of color-corrected lenses on each side of the stop.[3] Implicit in his design effort was that this was the fundamental design form (the necessary symmetry was preserved); the additional two elements were there only for color and aberration correction. Because the Mandler design suffers from oblique spherical, putting the gradient in the crown elements provides the best correction. At the time Pfisterer did his work, the material he needed was not available. Therefore he designed both the gradient and the dispersive properties of the glass. This work nicely illustrates the usefulness of determining the ideal gradient for the application, much like designing with model glasses in the early stages of a design. Figures 5-7 show the layout, ray fan and MTF plots for his design, scaled back to f/2.6.

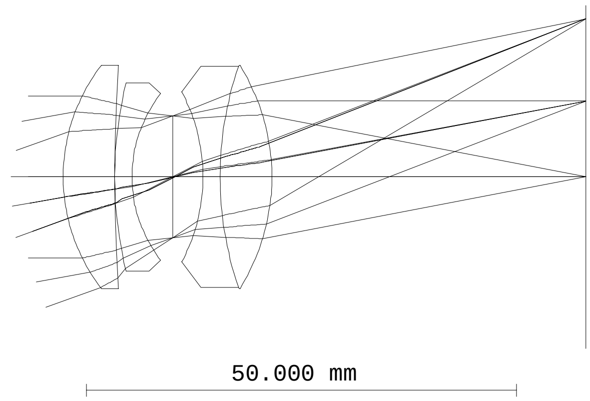

Figure 5. Pfisterer layout.

Pfisterer makes some comments in his paper that are particularly relevant to our assertion that a GRADIUM lens is not strictly equivalent to an asphere. He said:

"Perhaps the most striking feature of the aberration curves is the very small amount of residual oblique spherical aberration which usually afflicts the double Gauss design form. With so little flare in the field, I did not have to vignette nearly as much as usual to give an acceptable image. The consequence is that the relative illumination is approximately 15% higher at the edge of field than that of Mandler's design. (Since no commercially available optical design software is capable of calculating fifth-order aberration coefficients for GRIN elements, it is impossible to determine the gradient's effect on aberration correction and balance.) From experience with other axial GRIN designs, I have noted a general reduction in the amount of oblique spherical present even at large field angles."

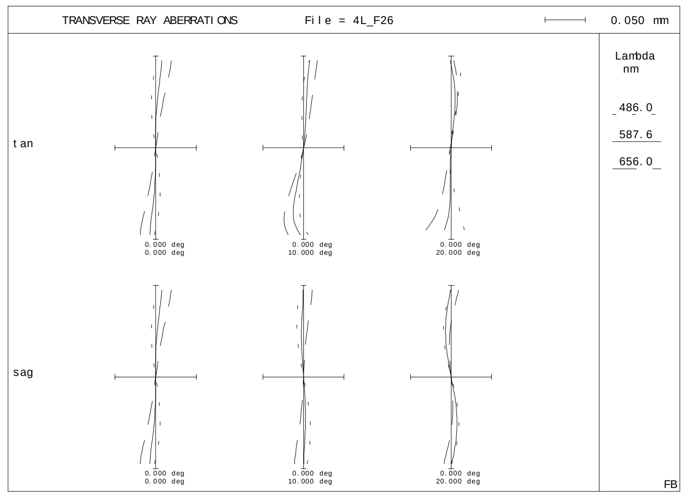

Figure 6. Pfisterer ray fan plots.

In his conclusions he added another relevant comment:

"As Sands[4] and others have pointed out, the inhomogeneous surface contribution of an axial gradient is functionally equivalent to that of an aspheric surface, at least to third order. The possibility that an aspheric version of this design exists was intriguing and so I spent some time looking at equivalent designs. By "equivalent" I refer to designs in which the inner homogeneous elements are identical to those of the GRIN design and the GRIN lenses are replaced by homogeneous aspheric lenses of arbitrary glass type.

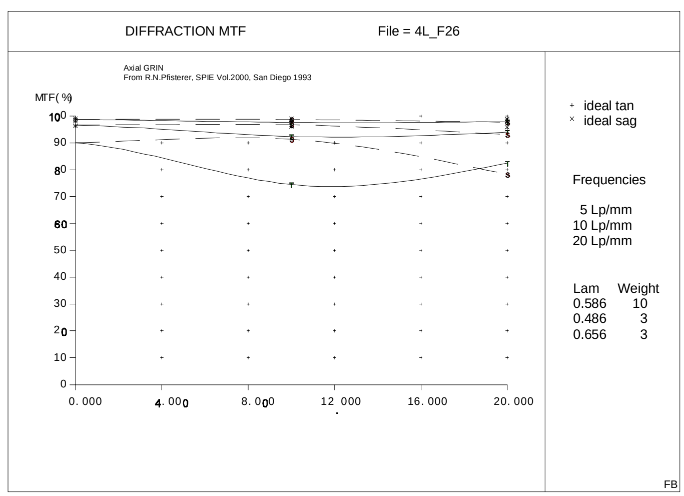

Figure 7. Pfisterer diffraction MTF versus field.

"After several failed attempts to replace GRIN elements with aspheric surfaces, I could not produce a design with comparable performance. The aspheric designs were plagued with uncontrollable chromatic aberrations, particularly axial color, and oblique spherical aberration."

2.3.3. "Pfisterer derivative" design

Fritz Blechinger has taken the Pfisterer design and worked it to a design that uses a real gradient material (therefore there was no freedom to design the gradient or the dispersion properties) and could, therefore, be built.[5] The G4LAK material was the obvious choice because it has base properties (index and Abbe number) that are similar to the synthetic gradient modeled by Pfisterer. This is a useful example of the final step in our design methodology—switching from the ideal to the real gradient.

In making the transition from the original Pfisterer design, which used synthetic gradients to a design using a real gradient, it was necessary to slow down the lens to f/2.6. This is because Pfisterer used crown-GRIN's with a very high index. This selection allowed him to achieve a good correction of the field curvature. Since the G4LaK GRADIUM does not provide such a high index, the field curvature in this "real-world" example is slightly worse. One interesting feature of this design is that only one axial gradient is used in this design, rather than two in the original Pfisterer design. The prescription is provided in the appendix. Comparing the ray aberration curves with the Pfisterer design shows that the performance of our "Pfisterer derivative" is comparable to the original Pfisterer design and clearly better than the Mandler reference design. Figures 8-10 give the layout, ray fans and MTF plots for this design.

We also note that because of the reduced speed of the lens, all the designs have essentially the same relative illumination. However, the Pfisterer derivative has the lowest distortion (-0.86% at a 20° field angle) unless some field curvature is sacrificed in the Pfisterer design (it can go below 0.2%). The total length is 66.7 mm, compared to 60.8 mm for the Pfisterer design and 69.7 mm for the Mandler design. Even with the slower f/# of this design, an attempt to replace the G4LAK element with an asphere (all varying all the radii in the lens) still produced a design with the same problems reported by Pfisterer. In addition, the size of the element makes actual production of such an asphere expensive.

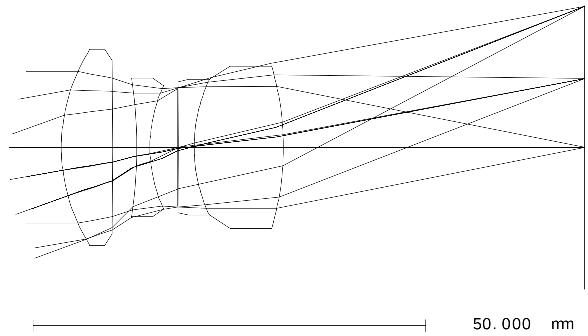

Figure 8. Pfisterer derivative layout.

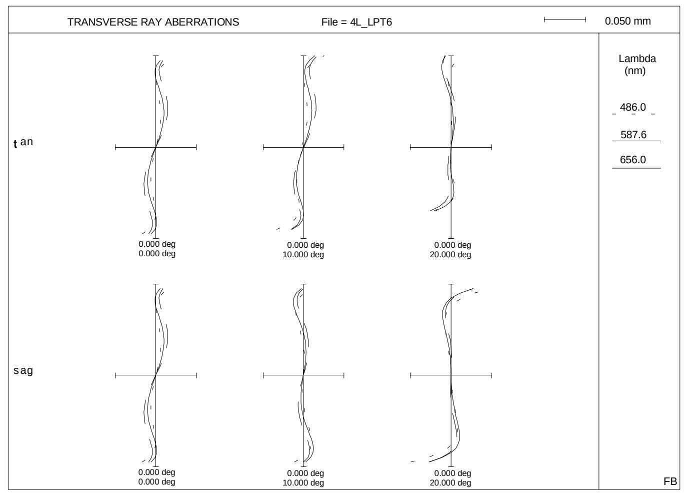

Figure 9. Pfisterer derivative ray fan plots.

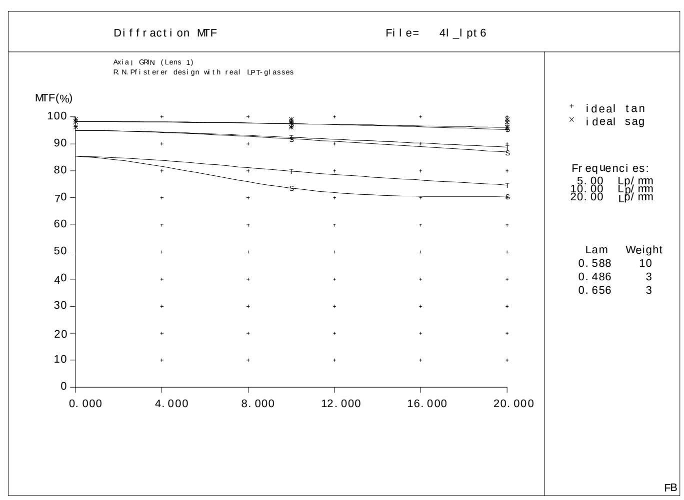

Figure 10. Pfisterer derivative diffraction MTF versus field.

2.4. Example: LightPath's Planar Perfecta lens

Recently, LightPath began production of a new 160 mm EFL f-Π lens that was initially designed by Ken Moore. This lens is somewhat unique in that it offers a larger field of view (±30.5°) and only uses two elements (instead of three or more), one of which is a GRADIUM lens. This lens is a classic example of how an axial gradient can be used to correct aberrations other than spherical. A layout plot is given in Figure 11.

In an f-Π lens, a relatively small beam is scanned across the lens. In the LightPath lens, the design form (lenses plus the location of the stop) are responsible for the gross distortion required by the f-Π condition (to about 1% compliance). However, the GRADIUM lens still provides the fine distortion correction required to meet the f-Π condition to within 0.1% and to flatten the field. At the simplest level, we realize that this happens because the scanning beam sees very different “lenses” at different field positions because the indices in the GRADIUM lens are different at each field position (although lens curvatures remain constant). There are other applications where putting a small beam through a GRADIUM lens allows this type of localized effect.

If the beam were expanded so that substantially the entire lens were illuminated, then the GRADIUM element would be correcting distortion and astigmatism, because the field curvature would be more of a macro correction based on vertex indices.

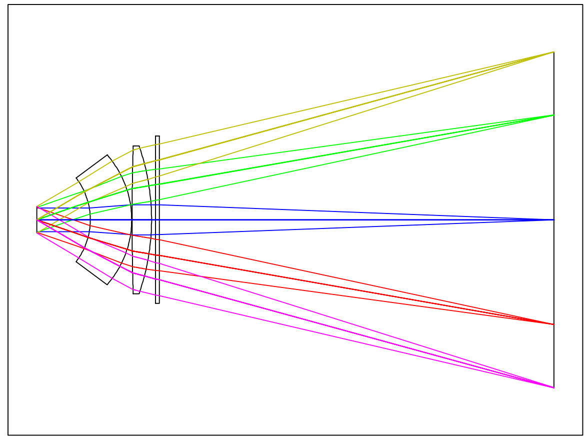

Figure 11. Layout of Planar Perfecta lens.

3. TOLERANCING AND PREPARING LENS PRINTS

3.1. Special Tolerancing Considerations

Tolerancing a GRADIUM lens is only slightly different than tolerancing a homogeneous lens. Generally, the most critical tolerancing parameters are wedge and −z. The different optical design software programs have different methods for implementing tolerances, but when tolerancing the GRADIUM element −z and GRADIUM tilt should be added to the normal list of tolerance parameters (such as mechanical wedge in the lens).

The −z tolerance that is routinely used at LightPath is ±50 Tm, although this tolerance can be tightened by a factor of two at many optical shops. The software tolerancing sensitivity to position may be increased at the extrema of the profile depending on what the program does when the z coordinate lies outside of the defined range. We have found that the standard −z tolerance is sufficient to detect sensitivity to the position of the lens vertex within the blank as well as the slight batch-to-batch profile deviations (typically the worst index error is • 0.0005).

GRADIUM tilt refers to the angular misalignment of the isoindex planes in the lens with respect to the optical axis of the lens. When the blank is prepared at LightPath, the isoindex planes and the high index (flat) surface are typically aligned to about 10 arc seconds. The total GRADIUM tilt will be given by the tilt in the blank plus the misalignment of the lens mechanical axis and the high-index surface normal. The design software should allow a means to do a small angle rotation of the GRADIUM coordinate system to mimic this effect.

The total GRADIUM tilt and wedge allowable in a design depend strongly on how the lens is used and its performance. When we consider the LightPath catalog singlet lenses, the lenses have been specified so that the RSS tilt/wedge error is about 1 arc minute. This is because many of these lenses are diffraction limited and allowing looser tolerances induces enough coma to significantly impact the lens performance. Because of the quality of the optics, LightPath has been unwilling to compromise on the manufacturing standards. Lenses that are not diffraction limited or because of their location in the system have more freedom can have looser specifications.

The process used to manufacture GRADIUM glass produces a glass that is free of bubbles, striae and is finely annealed. The glass good uniformity (homogeneity is an improper specification for the glass). Because of some of the processing details, there may be a small amount of residual radial gradient power in a blank. The power depends on the blank size and the profile, but in a 50 mm blank, full thickness, 2-3 waves of power might be present. The amount of power that is left in a finished lens, however, will be less than this, often negligible.

LightPath’s standard catalog singlet drawing notes are:

```
1. All dimensions in mm.
2. Surface figure measurements double pass at 633 nm.
3. Polish to test plate within power and
   irregularity indicated (fringes).
4. Surface quality scratch/dig 40-20. Laser grade finish.
5. Diameter +0.00 mm/ -0.025 mm.
6. Clear aperture central 90% of diameter.
7. GRADIUM profile offset tolerance +/- 0.050 mm.
8. Total measured optical wedge (mechanical + GRADIUM) <= 1 arcmin.
```

Typically, this requires that:
a. GRADIUM axial alignment in unfinished blank <=+/- 0.3 arcmin (as furnished).
b. Mechanical axis of lens to be within <=+/- 0.4 arcmin of the GRADIUM blank high index side surface normal.
c. Mechanical centration in finished lens <=+/- 0.7 arcmin.

9. Focal length tolerance at 546 nm +/- 1%.
10. Chamfer at 45 degrees the lesser of 5% of diameter or 0.35 mm.
11. Material names starting with G refer to GRADIUM(R) axial gradient glasses. Other glasses per MIL-G-174.
12. Design −z=_________mm (Engineering reference only).
13. Coating:____________.
14. Index at 70% clear aperture. Wavelength ___________nm.
    Front side n=__________

    Back side n=__________
15. First surface offset (delta z):_________mm.
16. Measure first surface offset from___________index side.

The tolerance in note 8a is controlled at LightPath's factory. Meeting the requirements of note 8 typically requires that the tolerances in notes 8b and 8c be held. It is important to note that a finished GRADIUM lens may have its diameter changed, but that it is not possible to mechanically center the lens after it has been produced. Thus, care must be taken when finishing the second side of the lens to keep the mechanical axes of both surfaces closely aligned. Note 8 reflects the measured optical performance that ultimately determines whether a lens passes the centration specification.

Note 14 reflects the fact that coatings on front and back surfaces should be adjusted for the different indices. When this is done, extremely high quality coatings are possible. The language of notes 12 and 15 is intended to force a distinction between the physical location of the lens vertex in a given blank and the design −z value. This will be discussed more in the next section.

3.2. Preparing a lens print

When ordering finished lenses from LightPath the lens print must contain several pieces of information. The standard notes given in § 3.1 describe most of the required information. What is important to realize is that the blank thickness defined in the optical design programs is a nominal thickness that is used to define the profile. LightPath does not stockpile finished blanks of different sizes. Rather, boules are manufactured, prisms are cut to check the profile, and then the location of the top surface with respect to the design profile z=0 coordinate is determined. (This is typically a fraction of a millimeter). With this information in hand, boules can be rethinned or a physical Δz value can be given to the finisher. What is important is to realize that the physical Δz must be decoupled from the design Δz. This is the reason for notes 12 and 15.

Figure 12 is an example of a properly prepared lens print. Note that the surface from which Δz is measured is clearly defined.

In the event that LightPath supplies only blanks (finished lenses are the preferred form for GRADIUM material sales), the customer will have to indicate the extreme positions in the design profile that are used, i.e. the lens uses material from z=3.1 to z=6.9 mm of G4SFN. Blanks will be prepared with a negotiated amount of material above and below these extreme locations. Then the customer's optical finishing shop will have the minimum amount of material to remove and every blank (for a particular part) will be optically identical.

4. CONCLUSIONS

LightPath's GRADIUM glass can be designed into optical systems to help improve performance or reduce complexity. Realizing that GRADIUM provides an aberration correction ability (but not the ability to defy first-order properties) and an investigation of the design help the designer determine how best to deploy GRADIUM lenses against a particular problem.

We have outlined a basic four-step process to determine where to place the GRADIUM lens and which profile and orientation to use. In essence this is a simple extension of techniques that are already used by designers. The results of using GRADIUM in a design can be dramatic, as illustrated by Pfisterer's earlier paper and Blechinger's realization of the design

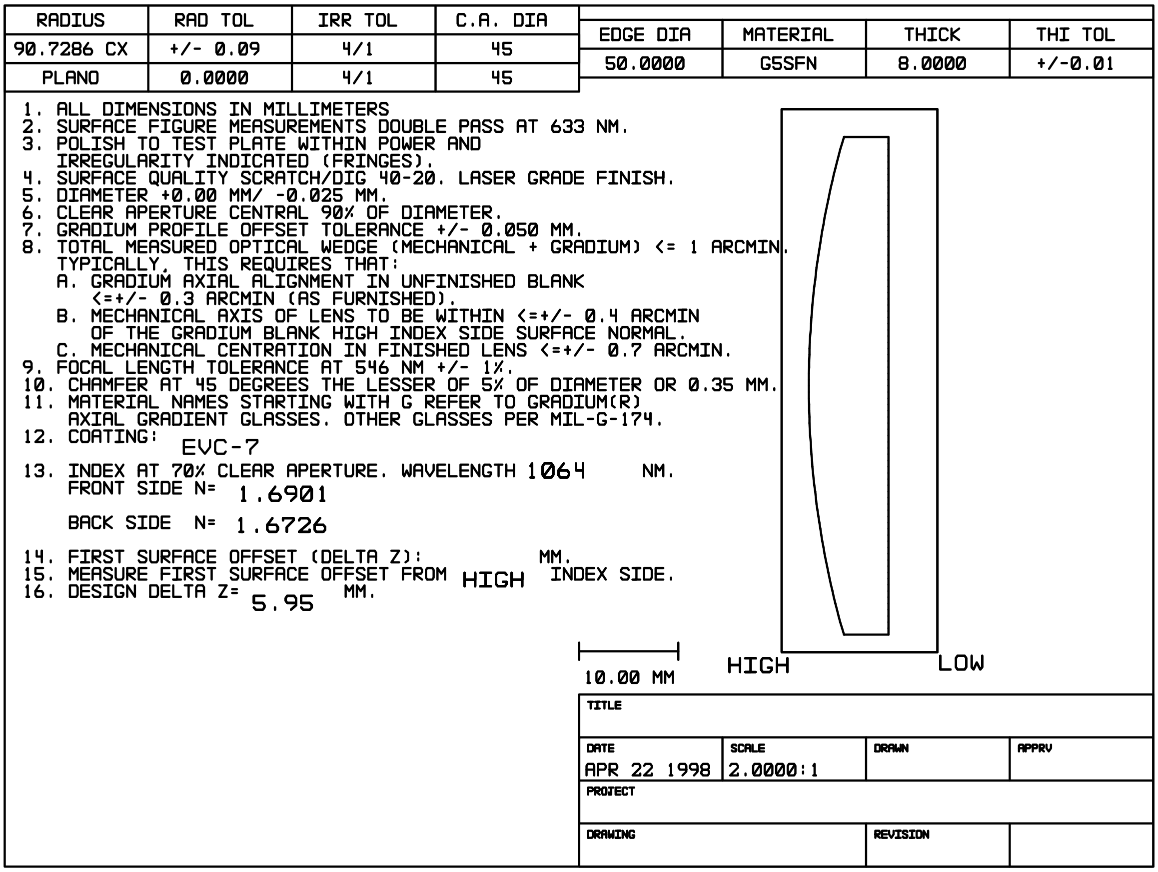

Figure 12. Example of a properly prepared lens print.

with real materials; the GRADIUM lens is better than the more complicated homogeneous lens. This result is typical of the results we have seen with GRADIUM systems, including some complicated systems that cannot be reported in this forum.

With the advances in GRADIUM glass manufacture and the improved design tools available, GRADIUM glass is a viable component of the modern optical designer's toolkit.

5. ACKNOWLEDGEMENTS

The authors would like to acknowledge the assistance and support rendered in the preparation of this manuscript and accompanying materials by Fritz Blechinger (Rodenstock Präzisionsoptik GmbH), Richard Pfisterer (Photon Engineering, LLC), Ken Moore (Focus Software, Inc.), Matthew Rimmer (Optical Research Associates), Doug Sinclair (Sinclair Optics) and David Freeman (Kidger Optics).

6. REFERENCES

1. W. Mandler, *Proc. SPIE* **237**, p. 222 ff., 1980.
2. L. G. Atkinson, et. al., "Design of a gradient-index photographic objective," *Applied Optics* **21** (6), pp. 993-998, 1982.
3. Richard N. Pfisterer, "Design of a 35mm photographic objective using axial GRIN materials," *Proc. SPIE* **2000**, 1993.
4. P. J. Sands, "Third-Order Aberrations of Inhomogeneous Lenses," *JOSA* **60** (11), pp. 1436-1443, 1970.
5. Fritz Blechinger, personal communication, 15 February 1998.

7. APPENDIX

7.1. GRADIUM Seidel coefficients-OSLO macro

```
cmd grinsei( void )
{
//computes Seidel aberrations for axial GRIN material
//rnp 17 november 1997

//paraxial raytrace values
  float ya, yb;
  float ua, ub;
  float uap;
  float ia, ib;
  float q;

  float n0;
  float n0p;
  float dndz, dnpdz;

  float mu;
  float opinv;
  float dt;
  float temp;

  float a1, a2, a3, a4, a5;
  float a1is, a2is, a3is, a4is, a5is, k;
  float a1it, a2it, a3it, a4it, a5it, ks;
  float sa3, cma3, ast3, ptz3, dis3;

  float sssa3, sscma3, ssast3, ssptz3, ssdis3;
  float sgsa3, sgcma3, sgast3, sgptz3, sgdis3;

//initialize sums
  sa3 = cma3 = ast3 = ptz3 = dis3 = 0.0;
  sssa3 = sscma3 = ssast3 = ssptz3 = ssdis3 = 0.0;
  sgsa3 = sgcma3 = sgast3 = sgptz3 = sgdis3 = 0.0;

//j is the surface number
  int j;

//dummy integer
  int i;

//clear buffer and trace paraxial a and b rays
  set_preference( output_text, off );
  ssbuf_reset();
  pxt all;
  set_preference( output_text, on );

//calculate scale factors
  mu = -1 / ( rn[ims][1] * ssb( ims+1, 2 ) );
  opinv = ssb( 2, 1 ) * rn[1][1] * ssb( 2, 5 ) -
          ssb( 2, 4 ) * rn[1][1] * ssb( 2, 2 );

//print out header
printf("\n*SEIDEL ABERRATIONS (including GRADIUMtm contributions)\n");
printf(" SRF      SA3          CMA3         AST3         PTZ3         DIS3\n");
//loop over surfaces
  for ( j = 1; j < ims; j++ ) {
    ya  = ssb( j+1, 1 );
    ua  = ssb( j,   2 );   //ua is after the current surface
```

```
    uap = ssb( j+1, 2 );
    ia  = ssb( j+1, 3 );
    yb  = ssb( j+1, 4 );
    ub  = ssb( j,   5 );
    ib  = ssb( j+1, 6 );

    n0 = rn[j-1][1];
    n0p = rn[j][1];
    dndz = dnpdz = 0.0;
    k = 0.0;
    ks = 0.0;
    a1is = a2is = a3is = a4is = a5is = 0.0;
    a1it = a2it = a3it = a4it = a5it = 0.0;

    set_preference( output_text, off );
//if Gradium, then get indices at surface poles
//prior surface
    if ( gdt[j-1] == 11 ) {
      i = sbrow();
      grad_index_value( j-1, 1, 0, 0, th[j-1] );
      n0 = ssb( i, 4 );
      dt = dth[ j-1 ];
      grad_index_value( j-1, 1, 0, 0, th[j-1]-dt );
      temp = ssb( i+1, 4 );
      dndz = ( n0 - temp ) / dt;
    }
//current surface
    if ( gdt[j] == 11 ) {
      i = sbrow();
      grad_index_value( j, 1, 0, 0, 0 );
      n0p = ssb( i, 4 );
      dt = dth[ j ];
      grad_index_value( j, 1, 0, 0, dt );
      temp = ssb( i+1, 4 );
      dnpdz = ( temp - n0p ) / dt;
    }
    set_preference( output_text, on );

//homogeneous surface contributions
    a1 = mu * 0.5 * n0 * ( n0/n0p - 1.0 ) * ya * ia * ia * ( ia + uap );

    q = 0.0;
    if ( ia != 0.0 )
      q = ib / ia;

    a2 = q * a1;
    a3 = q * a2;
    a4 = mu * 0.5 * opinv *opinv * cv[j] * ( 1.0/n0p - 1.0/n0 );
    a5 = q * ( a3 + a4 );

//inhomogeneous surface contributions
    if ( gdt[j-1] == 11 || gdt[j] == 11 ) {
      k = mu * (- 0.5) * cv[j] * cv[j] * ( dnpdz - dndz );
      a1is = k * ya * ya * ya * ya;
      a2is = k * ya * ya * ya * yb;
      a3is = k * ya * ya * yb * yb;
      a4is = 0.0;
      a5is = k * ya * yb * yb * yb;
    }

//inhomogeneous transfer contributions
//  assume that grin is linear axial, do integral in closed form
  if ( gdt[j-1] == 11 ) {
```

```
    set_preference( output_text, off );
    i = sbrow();
    grad_index_value( j-1, 1, 0, 0, 0 );
    n0 = ssb( i, 4 );
    grad_index_value( j-1, 1, 0, 0, th[j-1] );
    n0p = ssb( i+1, 4 );
    set_preference( output_text, on );

    dndz = ( n0p - n0 ) / th[j-1];

    ks = ( ssb( j+1, 1 )/(n0p*n0p) ) - ( ssb( j, 1 )/(n0*n0) );
    ks += n0 * ua * ( 1/(n0p*n0p) - 1/(n0*n0) ) / ( 2 * dndz );

    a1it = mu * 0.5 * ( n0 * ua )**3 * ks;
    a2it = mu * 0.5 * n0**3 * ua**2 * ub * ks;
    a3it = mu * 0.5 * n0**3 * ua * ub**2 * ks;
    a4it = 0.0;
    a5it = mu * 0.5 * n0**3 * ub**3 * ks;
  }

//print it all out
    printf(" %2d  %f  %f  %f  %f  %f \n", j, a1, a2, a3, a4, a5 );
    if ( fabs( k ) > 1e-30 )
      printf(" IS  %f  %f  %f  %f  %f \n", a1is, a2is, a3is, a4is, a5is );
    if ( fabs( ks ) > 1e-30 )
      printf(" IT  %f  %f  %f  %f  %f \n", a1it, a2it, a3it, a4it, a5it );

    if ( gltyp[j] == 1 )
      printf("\n");

//sum homogeneous surface contributions
    sssa3 += a1;
    sscma3 += a2;
    ssast3 += a3;
    ssptz3 += a4;
    ssdis3 += a5;

//sum inhomogeneous gradient contributions
    sgsa3 += a1is + a1it;
    sgcma3 += a2is + a2it;
    sgast3 += a3is + a3it;
    sgptz3 += a4is + a4it;
    sgdis3 += a5is + a5it;

    sa3  += a1 + a1is + a1it;
    cma3 += a2 + a2is + a2it;
    ast3 += a3 + a3is + a3it;
    ptz3 += a4 + a4is + a4it;
    dis3 += a5 + a5is + a5it;

  }  //end of surface loop

printf(" SUM %f  %f  %f  %f  %f \n\n", sa3, cma3, ast3, ptz3, dis3 );
printf(" HOM %f  %f  %f  %f  %f \n", sssa3, sscma3, ssast3, ssptz3, ssdis3 );
printf(" GRN %f  %f  %f  %f  %f \n", sgsa3, sgcma3, sgast3, sgptz3, sgdis3 );
}
```

7.2. GRADIUM coefficients

The Pfisterer derivative was designed using a preliminary G4LAK profile definition and dispersion modeling. The modeling issues are discussed in “Current developments in GRADIUM® technology,” also presented in this session. For completeness, here are the coefficients that were used:

```
G4LAKN     Z-MAX =  13.9312

Profile Coefficients:
N0= 1.7383803E+00
N1=-2.7823900E-02
N2= 2.6596027E-01
N3=-7.1780327E+00
N4= 6.3229961E+01
N5=-3.0798327E+02
N6= 9.0872128E+02
N7=-1.6864754E+03
N8= 1.9809272E+03
N9=-1.4296996E+03
N10=5.7883131E+02
N11=-1.0067746E+02

Sellmeier dispersion model coefficients:
K-coefficients:
```

0.00522664	0.0206983	-0.00450304	0.006873	0	0
0.0472841	0.0429402	-0.00724884	-0.0445419	0	0
0.988601	0.057962	0.0941671	0.152672	0	0

L-coefficients:

0.0421634	0	0	0	0	0
0.0368588	0	0	0	0	0
110.0	0.0	0.0	0.0	0.0	0.0

7.3. Double Gauss prescription-Pfisterer Derivative

The following parameters help to understand the listings:

```
R.N.Pfisterer design with real LPT-glasses

 Wavelength :     0.58760    0.48600    0.65600
 Weight     :          10          3          3
 REF =  1
```

#	TYPE	RADIUS	DISTANCE	GLASS	INDEX	X-APE	Y-APE	CP	DP
1	SI	23.23606	6.50000	G4LAKN	1.717414	0.00	12.50*	0	0
		–Z=	5.05705						
2	S	-909.97047	3.12330		1.000000	0.00	11.06	0	0
3	S	-55.46133	1.66667	SF2	1.647685	0.00	8.81	0	0
4	S	18.78932	3.53743		1.000000	0.00	7.86	0	0
STO	S	Infinity	0.09994		1.000000	0.00	7.50	0	0
6	S	-548.36394	1.99995	SF1	1.717355	0.00	8.33*	0	0
7	S	20.36501	11.36424	LASFN31	1.880665	0.00	8.68	0	0
8	S	-36.56109	38.37492		1.000000	0.00	10.32	0	0
IMG	S	Infinity			1.000000	0.00	18.06	0	0

An * denotes a hard aperture used to vignette the system.